



# Advanced stability analysis and design of a new Danube archbridge

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# Subject of the lecture

Buckling of steel tied arch

Buckling of orthotropic steel plates

**ANALYSIS – DESIGN METHODS – APPLICATION**



# Contents

## About the bridge

## Global buckling of tied arch

- Experimental buckling analysis
- Evaluation of classical and advanced design methods
- Application

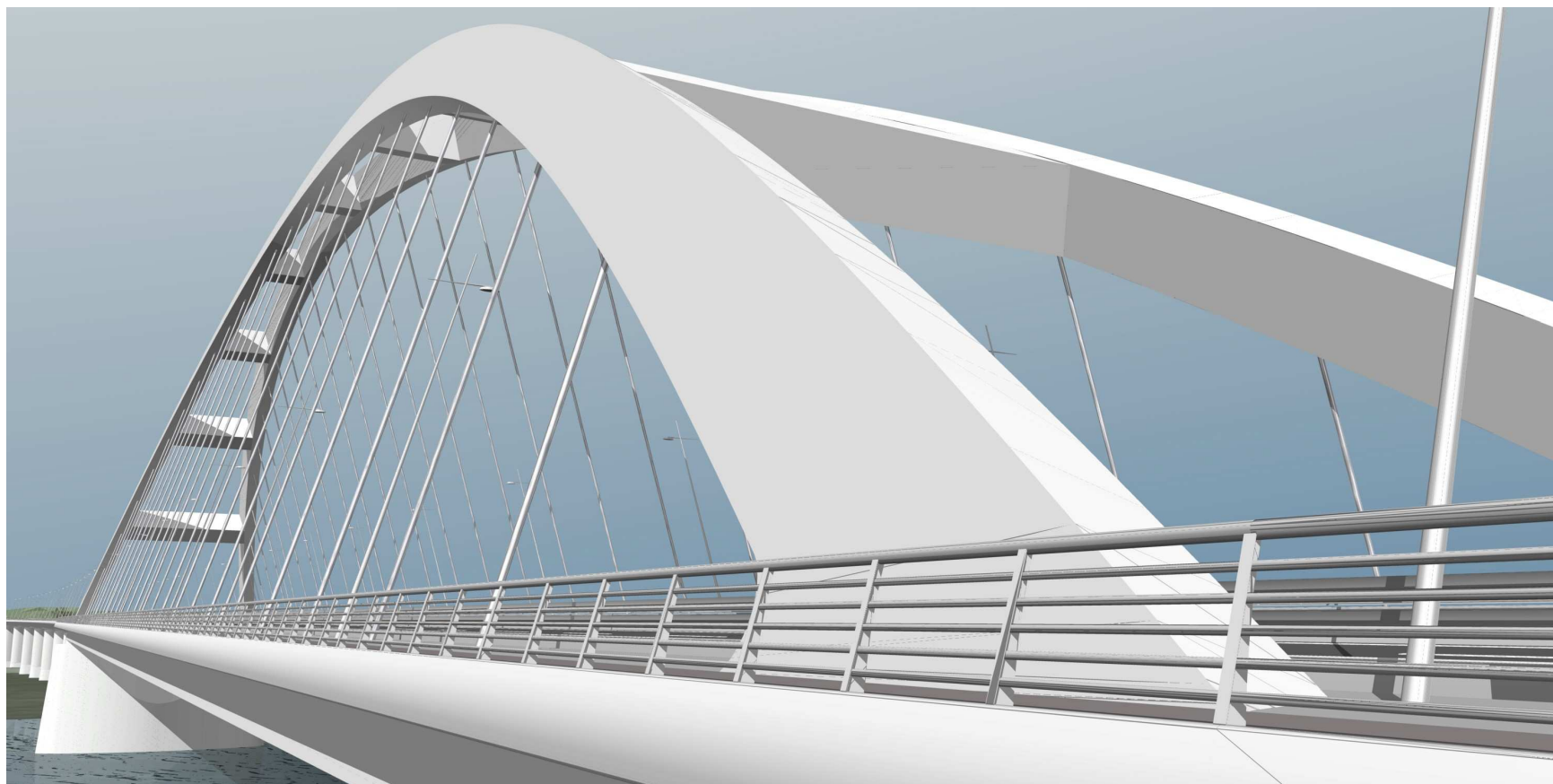
## Orthotropic plate buckling

- Overview of design methods
- FE simulation based stability analysis and design
- Application

## Concluding remarks



# Dunaújváros Danube bridge





## Location





## Geometry

Total length of the bridge:	1 780 m
Main span; tied arch bridge:	307.8 m
arch height:	48 m
steel box:	2 x 3.8 m



## Current stage



Webcam:

<http://www.dunaujhid.hu/webcam.html>



## Tasks of the Department

Preliminary phase: advisor for designer

Design phase: research on design methods  
model test – arch stability  
wind tunnel test on section model  
analysis and design  
stability, fatigue,  
earthquake, aerodynamic

Construction phase: erection method  
structural design for erection





## Model test on arch stability



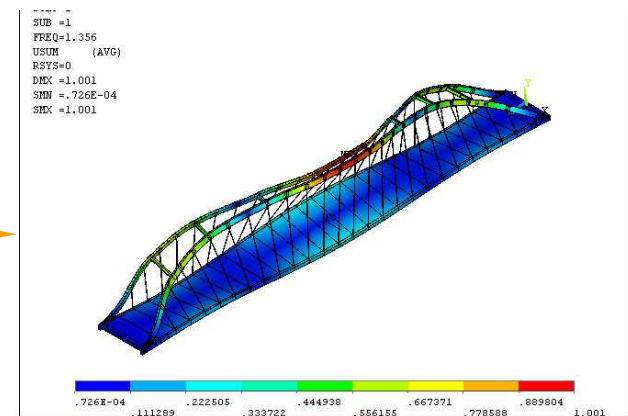
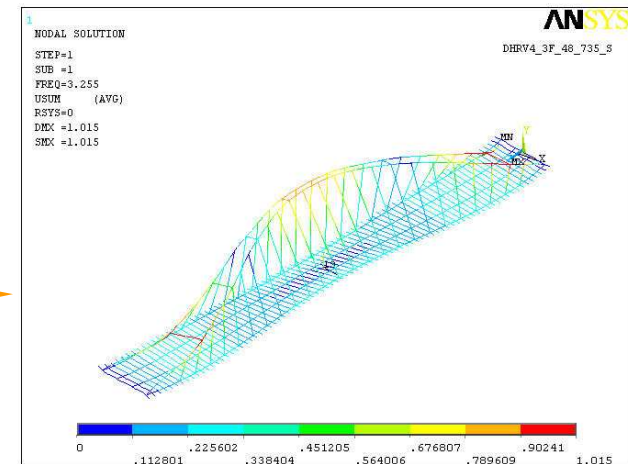


## Purpose

### Experimental test

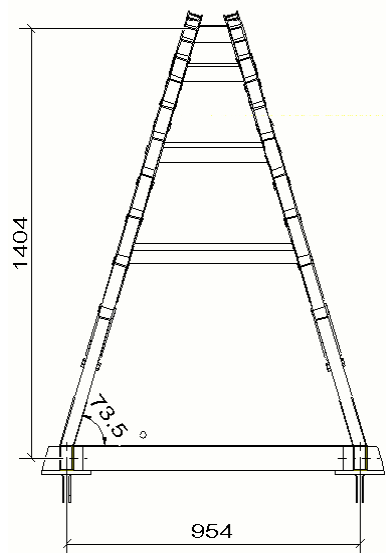
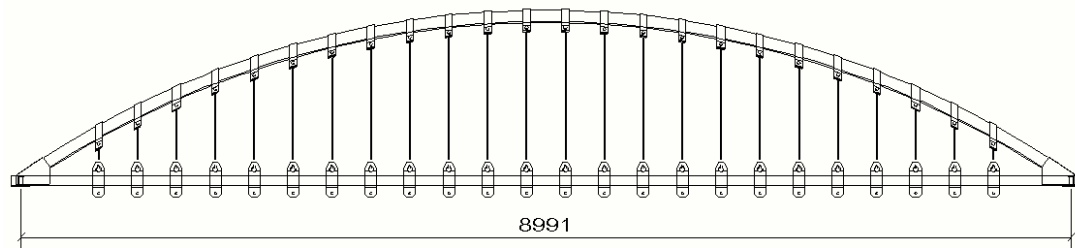


- 1) check the safety of the standard design methods
- 2) verify advanced numerical model and calibrate imperfection sizes





## Bridge model M=1:34

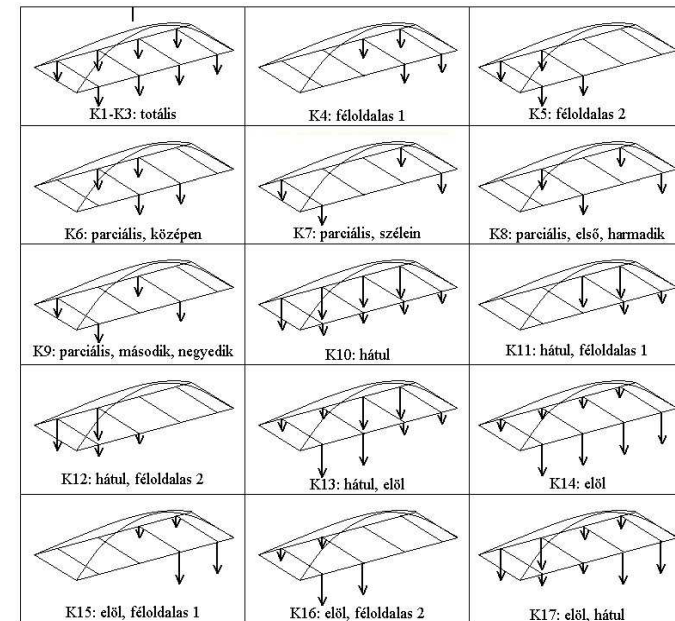


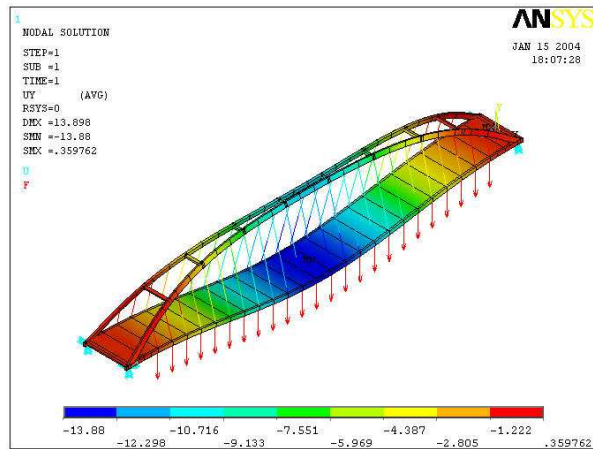


## Loading system

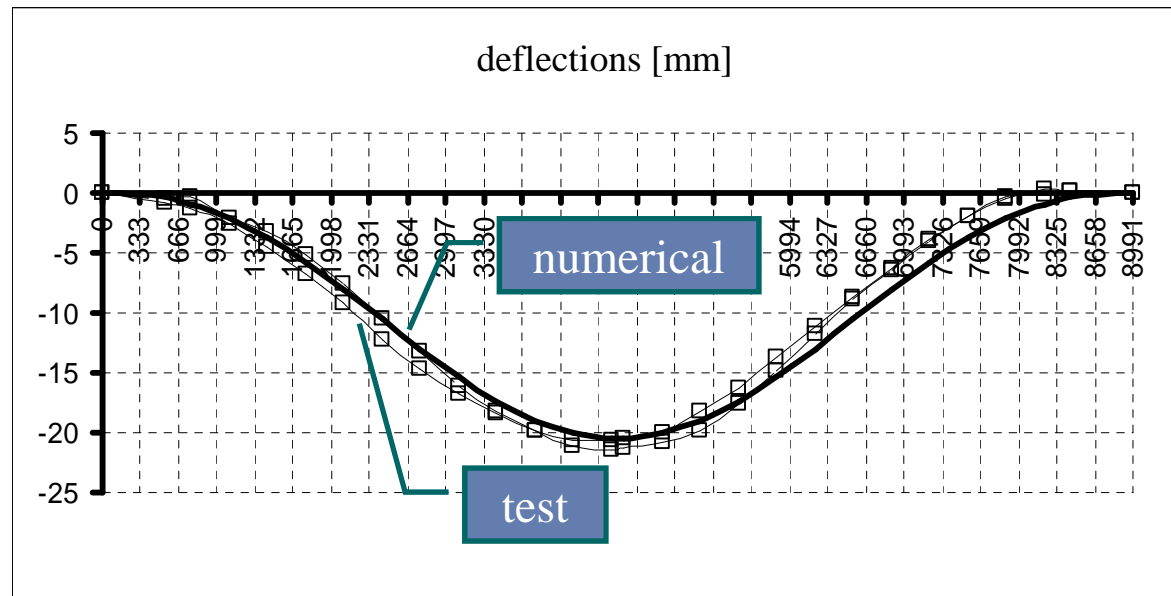
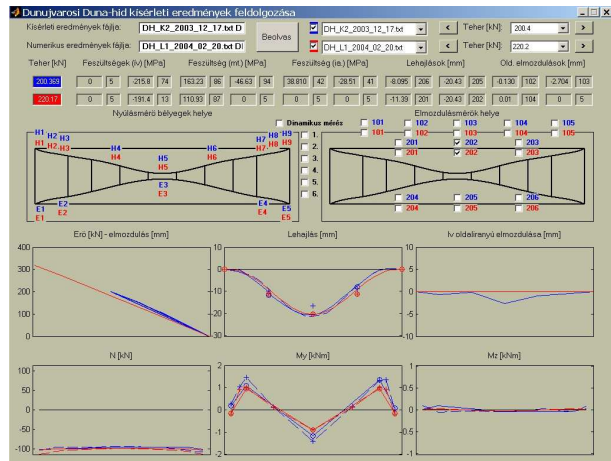


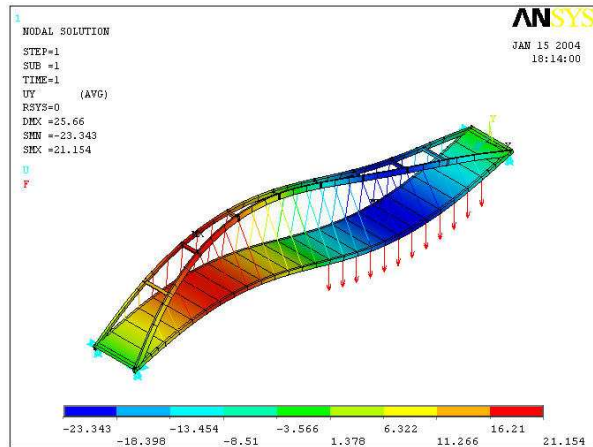
### 15 load cases



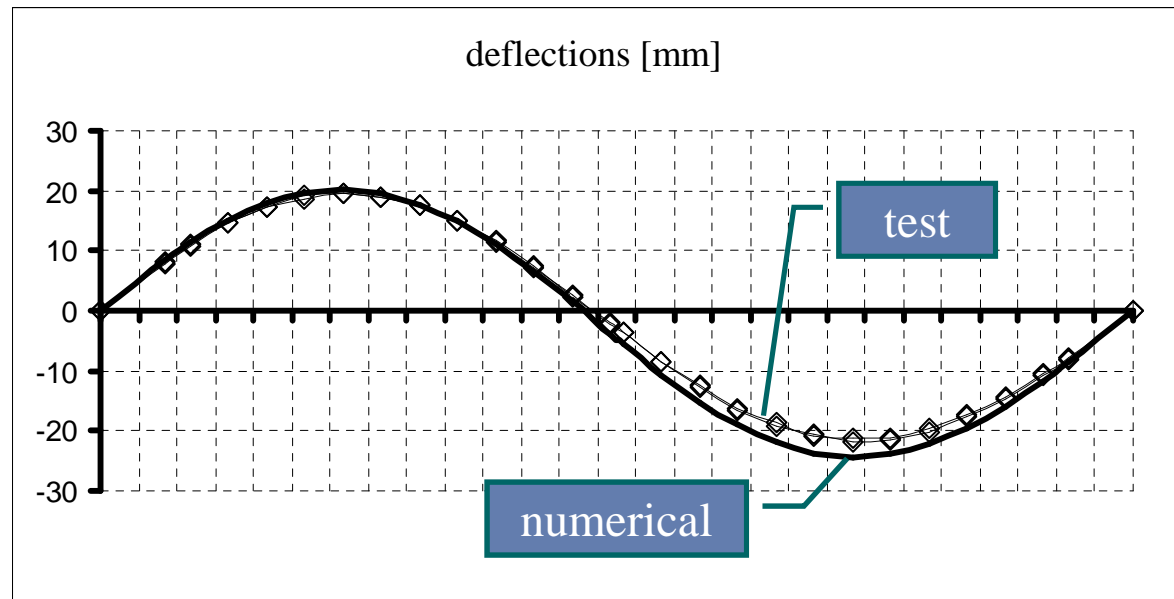
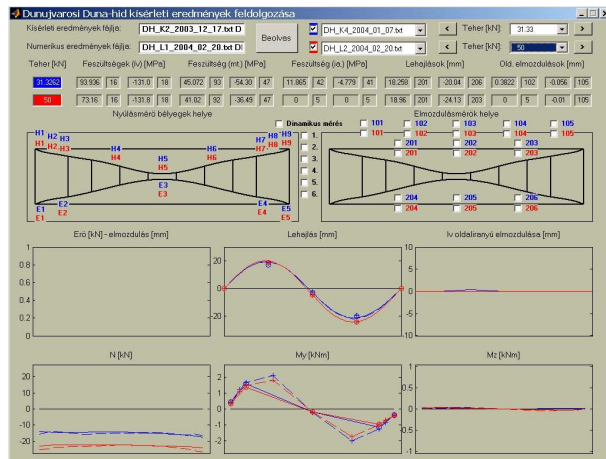


Total loading:  $\Sigma q=220$  kN  
 Self-weight + 75 x 40 t trucks



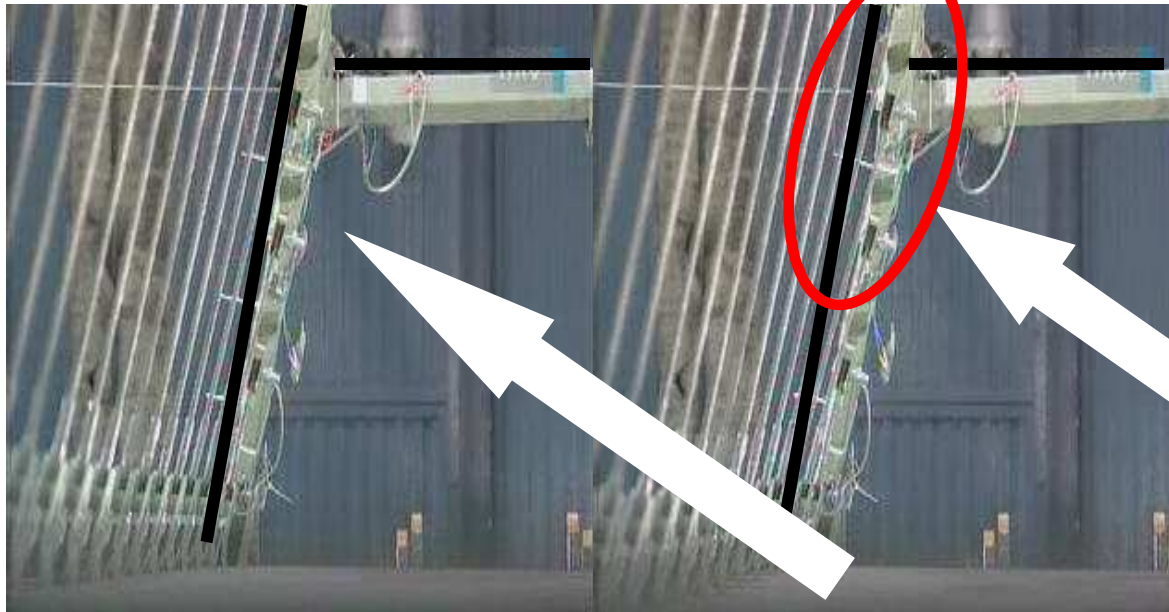


partial half-sided loading:  $\Sigma q=50$  kN



## Failure test - 1

total loading: 320 kN out-of-plane buckling



out-of-plane buckling of the arch

local plate buckling









## Failure test - 2

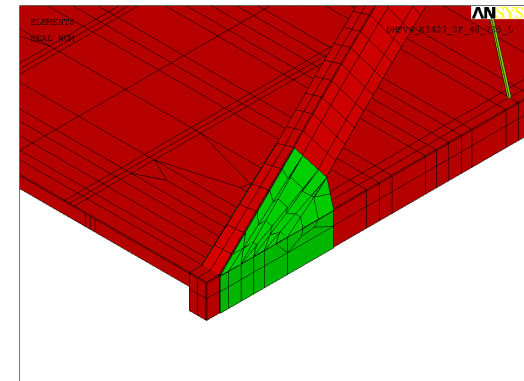
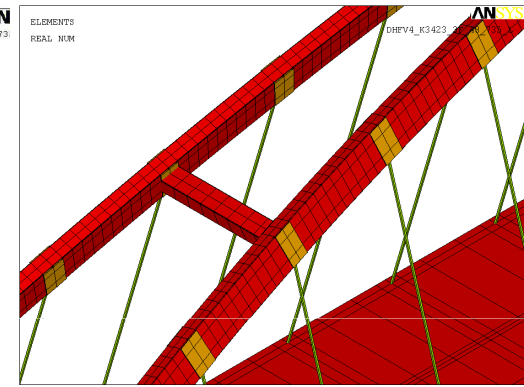
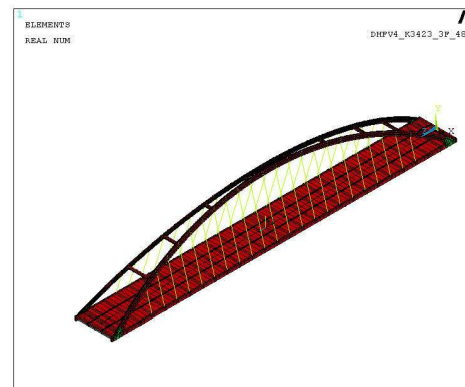
half-sided loading: 110 kN



in-plane buckling of the arch

# Numerical model

Model data	Ansys beam model	Ansys shell model
Element type	BEAM44 LINK10	SHELL181 LINK10
number of elements	~6 000	~17 000
number of nodes	~12 000	~17 000



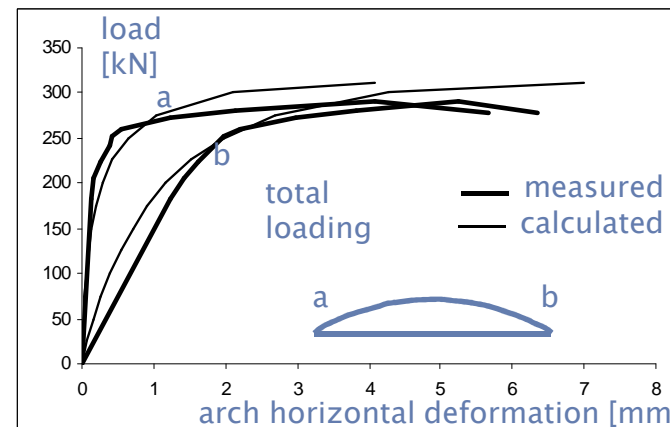
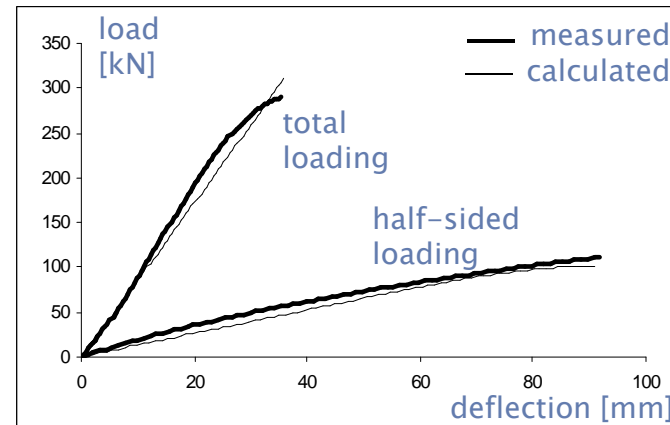
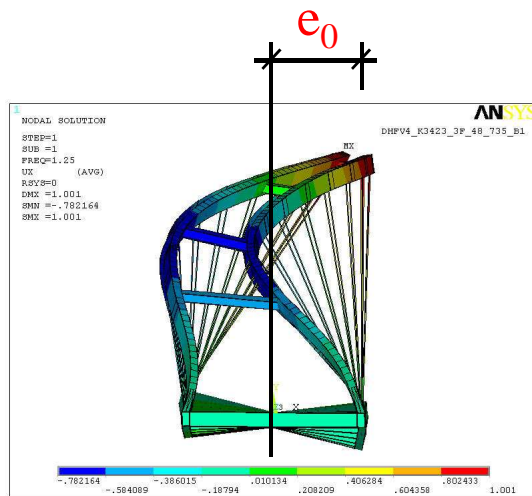
Analysis	
<b>Linear</b>	material and geometrical linearity
<b>Instability</b>	Block Lanczos buckling analysis
<b>Geometrically nonlinear</b>	geometrical nonlinearity, imperfect model
<b>Virtual experiment</b>	material and geometrical nonlinearity, imperfect model

# Verification

material and geometrical nonlinearity

imperfection:  $e_0 = 2 \text{ mm}$

on the half side of the model  $\Rightarrow$   
 non-symmetrical behaviour



# Design methods

Hungarian Standard: (1)  $\frac{N}{N_e} + \psi_y \frac{M_y}{M_{e,y}} + \psi_z \frac{M_z}{M_{e,z}} \leq 1$  strength check

HS

+

(2)  $\frac{N}{N_{ke,z}} \leq 1$  buckling check

Japanese Standard: (3)  $\frac{N}{N_{ke,z}} + \psi_y \frac{M_y}{M_{e,y}} + \psi_z \frac{M_z}{M_{e,z}} \leq 1$  linear interaction

JSHB

Eurocode:

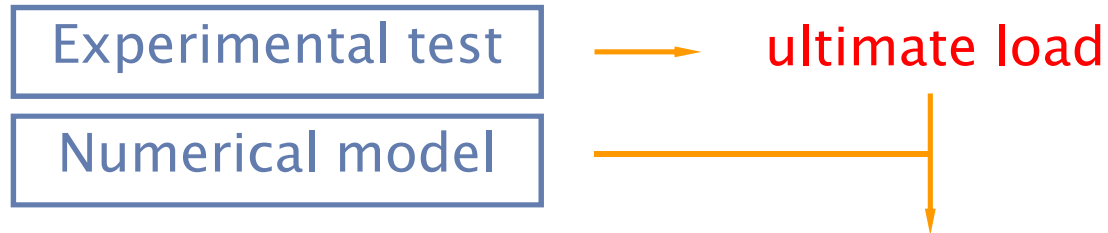
(4)  $\frac{N}{N_{ke,y}} + k_{yy} \frac{M_y}{M_{e,y}} + k_{yz} \frac{M_z}{M_{e,z}} \leq 1$

EC3

(5)  $\frac{N}{N_{ke,z}} + k_{zy} \frac{M_y}{M_{e,y}} + k_{zz} \frac{M_z}{M_{e,z}} \leq 1$

} combined interaction

## Solution method



Internal forces:  $N$ ,  $M_y$ ,  $M_z$  from analyses:

1. linear + second order modification factor
2. geometrically nonlinear – equivalent imperfection 1
3. geometrically nonlinear – equivalent imperfection 2

standard ultimate load ↔ experimental ultimate load

# Equivalent imperfection size 1

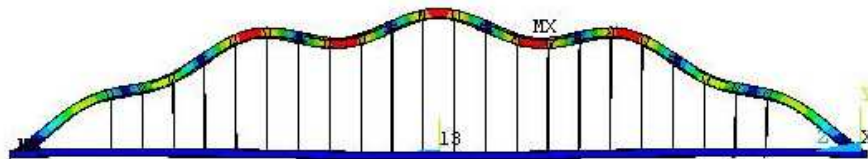
Eurocode 3 – Part 1.1: shape of the elastic critical buckling mode:  $\eta_{cr}$

$$e_{0,d} = \alpha(\bar{\lambda} - 0.2) \frac{M_{Rk}}{N_{Rk}} \frac{1 - \chi \bar{\lambda}^2}{1 - \chi \bar{\lambda}^2} \gamma_{M1}$$

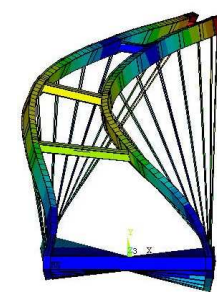
$$\eta_{init} = \frac{e_{0,d}}{\lambda^2} \frac{N_{Rk}}{EI \eta''_{cr,max}} \eta_{cr}$$

second order analysis

in-plane buckling mode:



out-of-plane buckling mode:



In-plane	Out-of-plane
3.44 mm	17.30 mm

## Equivalent imperfection size 2

Eurocode 3 – Part 2:

(Design of bridges)

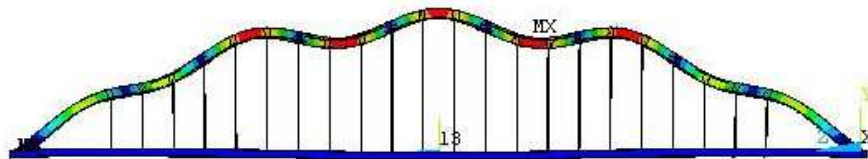
$$\eta_{0,z} = \frac{l}{500}$$

in-plane

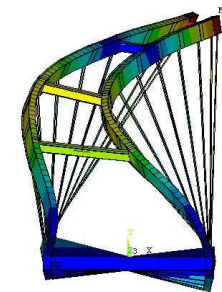
$$\eta_{0,y} = \frac{l}{250}$$

out-of-plane

in-plane buckling mode:



out-of-plane buckling mode:



In-plane	Out-of-plane
17.98 mm	35.96 mm

# Comparison of classical design methods

	total load	half-sided load
HS	2.25	3.06
JSHB	3.07	3.28
EC3	2.20	1.87

experimental ultimate load / standard ultimate load





## Comparison of Eurocode approaches

	total load	half-sided load
EC3 – linear	2.20	1.87
EC3 – eqv. geom. imp. 1	1.45	1.84
EC3 – eqv. geom. imp. 2	2.29	2.15

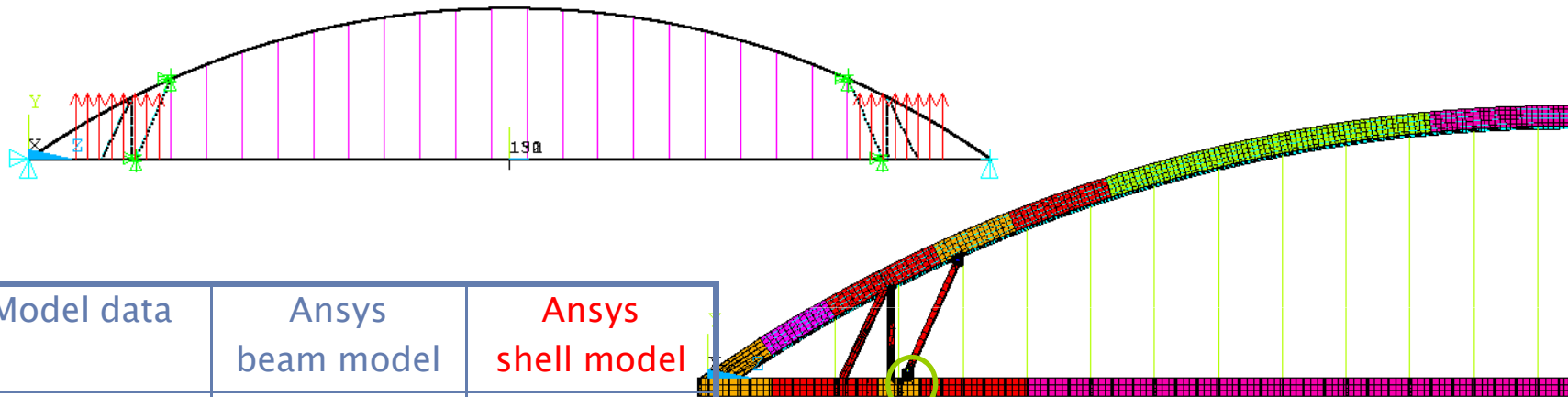
experimental ultimate load / standard ultimate load



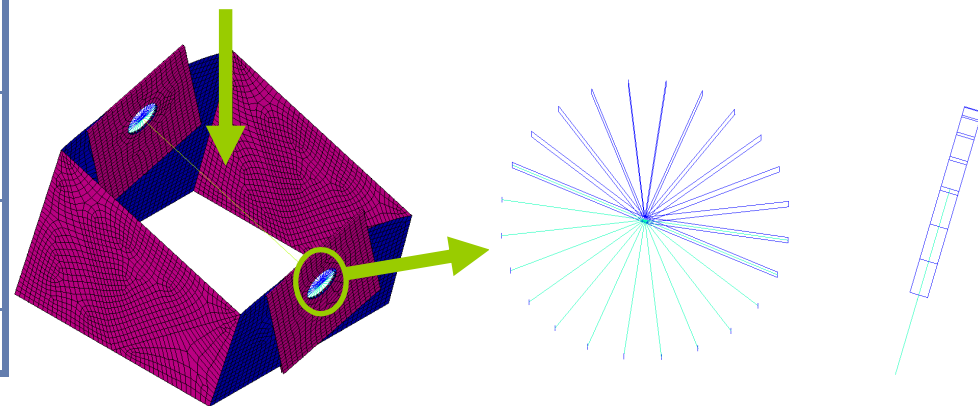
## Arch bridge erection



# Finite element model



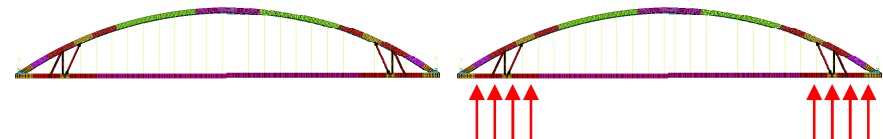
Model data	Ansys beam model	Ansys shell model
Element type	BEAM44 LINK10	SHELL181 LINK10
number of elements	~29 000	~170 000
number of nodes	~57 000	~170 000
DOF	~340 000	~1 000 000





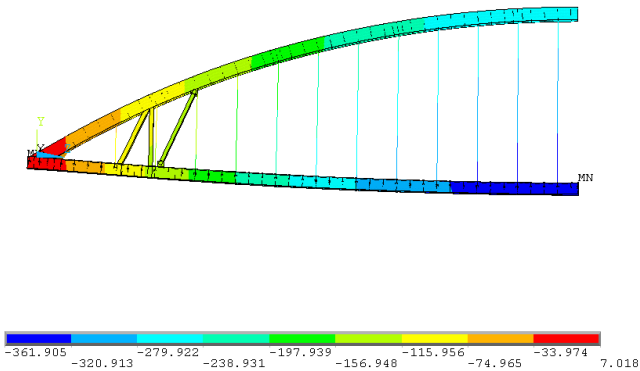
## Erection phases

1. Bridge is on the riverbank on a rack system
2. The cables are stressed to the self weight
3. Additional bars are built in the bridge
4. The bridge is palced on barges

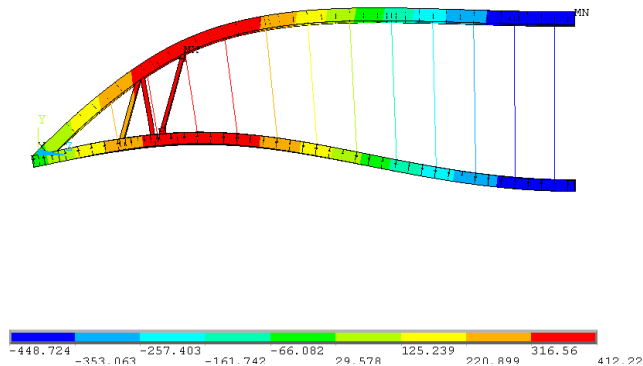




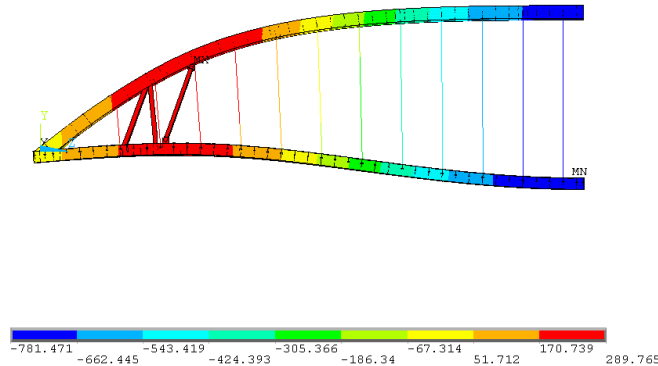
# Stress analysis



1. Load case: self-weight



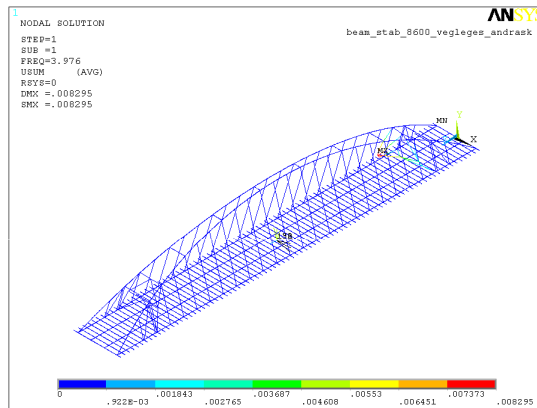
2. Load case: ship reaction forces



3. Load case: 1. load case + 2. load case

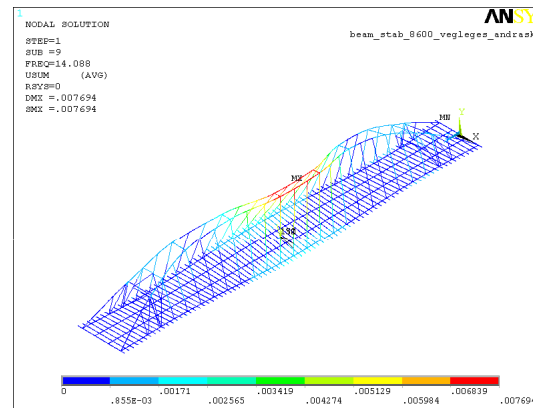
# Instability analysis

Stiffening bar buckling



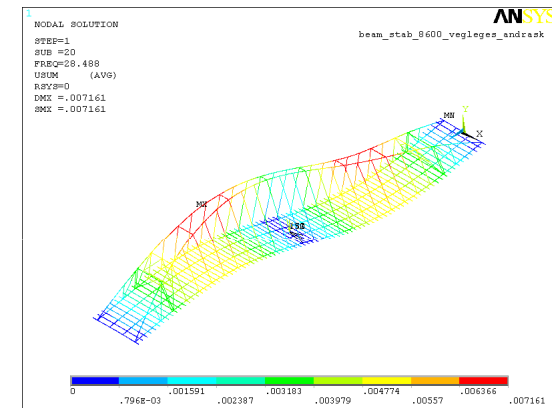
$$\alpha_{cr} = 3.97$$

Out-of-plane buckling



$$\alpha_{cr} = 14.088$$

In-plane buckling



$$\alpha_{cr} = 28.488$$

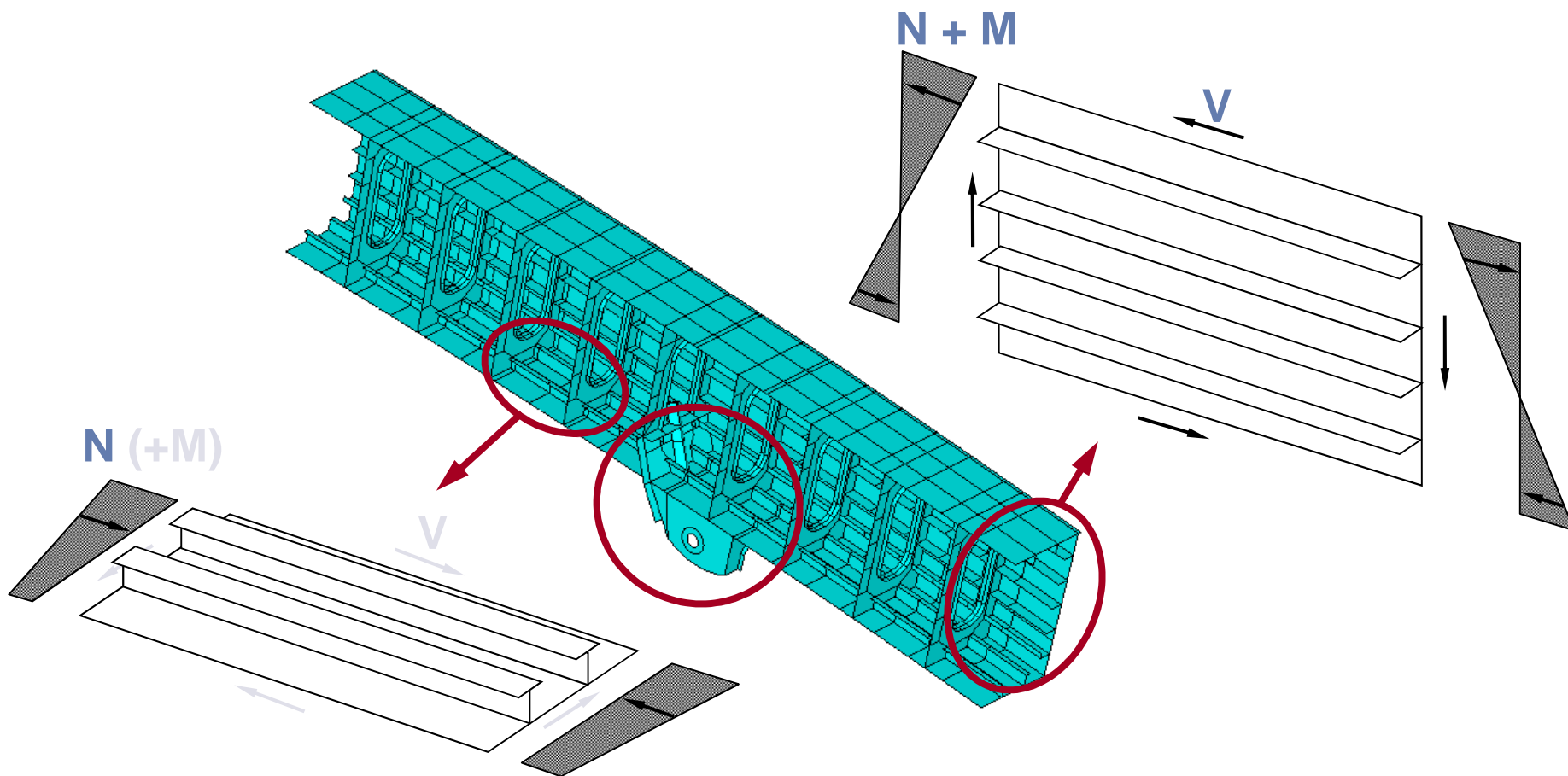


# Orthotropic plate buckling





# Orthotropic plates





## Design methods – Hungarian Standard

- allowable stress design

$$f_y = 460 \text{ MPa} \xrightarrow{\text{factor } \sim 1.47} \sigma_e = 300 \text{ MPa}$$

- dominantly compressed stiffened plates or plate parts  
→ (1) buckling of fictive column stub
- stiffened plate subject to complex stress field  
→ (2) orthotropic plate check
- irregular configuration and stress field – ??? no rule given  
→ (3) generalized plate check



## (1) Buckling of fictive column stub

- fictive column = stiffener + adjacent plating
- column slenderness ( $\lambda$ ) and reduction factor ( $\phi$ )

a) flexural buckling

$$\lambda_1 = L_{kt} \cdot \sqrt{\frac{A_b}{I_b + b_e \cdot \frac{t_f^3}{11} \cdot \left( 2 \cdot \frac{L_{kt}^2}{4 \cdot b_s^2} + \frac{L_{kt}^4}{16 \cdot b_s^4} \right)}}$$

$$\phi_1 = \beta - \sqrt{\beta^2 - \frac{1}{(\lambda_1 / \lambda_E)^2}}$$

b) torsional buckling

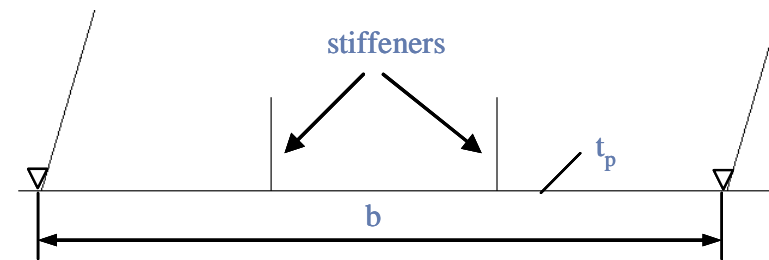
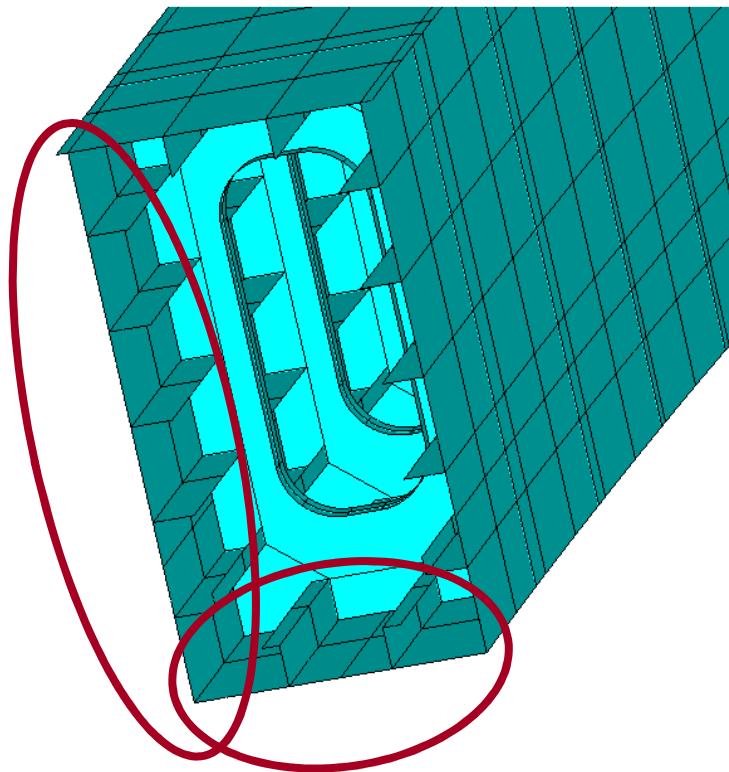
$$\lambda_2 = L_{kt} \cdot \sqrt{\frac{I_u + I_\eta}{0.04 \cdot L_{kt}^2 I_T + I_\eta \cdot h^2}}$$

$$\phi_2 = \left( \frac{1}{1 + (\lambda_2 / \lambda_E)^5} \right)^{0.4}$$

$\phi \cdot \sigma_e$  (allowable stress)

$$\cdot \text{check: } \frac{N_b}{A_b} \leq \phi \cdot \sigma_e$$

## Actual calculations on the Danube bridge



Case Nr.	$t_p$ [mm]	$b$ [m]	$a$ [m]	stiffener
1	40	2	4.56	2 x 280-22
2	30	3.8	4.56	5 x 280-22
3	50	2	2.125	2 x T270-150-22
4	20	3.8	3.9	5 x 280-22
5	16	3.8	3.86	5 x 280-22
6	20	2	3.86	2 x 280-22

$t_p$  – plate thickness;  $b$  – plate width;  $a$  – plate length between transverse stiffeners or diaphragms



## (2) Orthotropic plate subject to complex stress field

- plate-type behaviour
- plate slenderness ( $\lambda_0$ ) and reduction factor ( $\phi_b$ )

$$\lambda_0 = \frac{3.3}{\sqrt{k_{red}}} \frac{b}{t}$$



$\phi_b$  reduction factor for plates

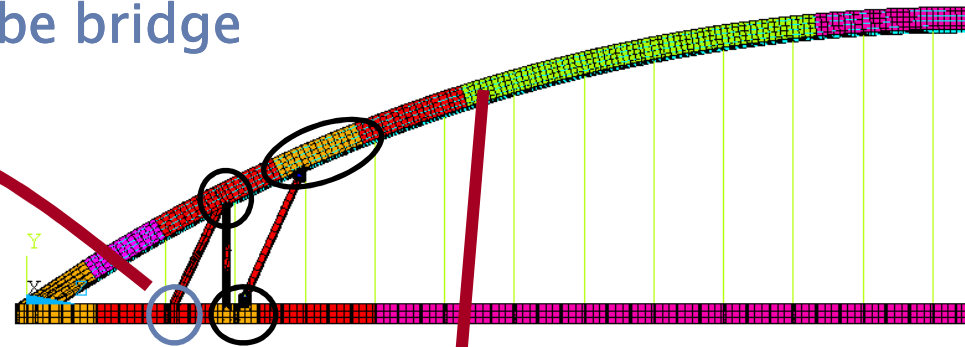
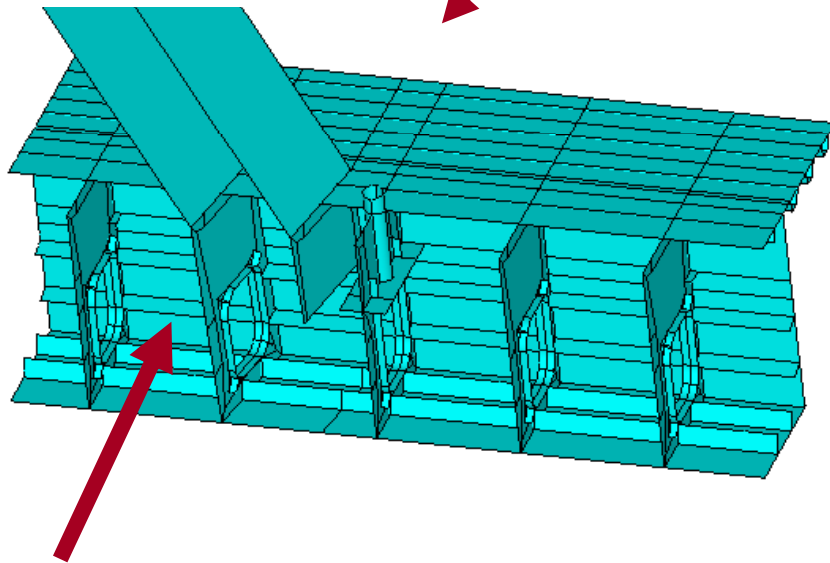
$k_{red}$ : buckling coefficient, e.g. Klöppel–Scheer–Möller (overall plate buckling of horizontally and longitudinally stiffened plates)

$$\cdot \text{check: } \sigma_{red} = \sqrt{\sigma^2 + 3\tau^2} \leq \phi_b \cdot \sigma_e$$



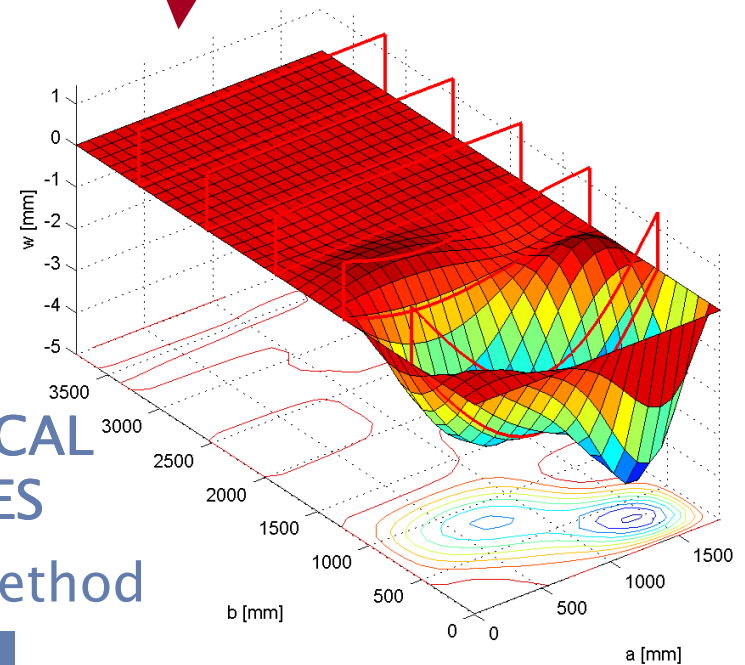
## Actual calculations on the Danube bridge

B) SUBMODELS  
– FEM

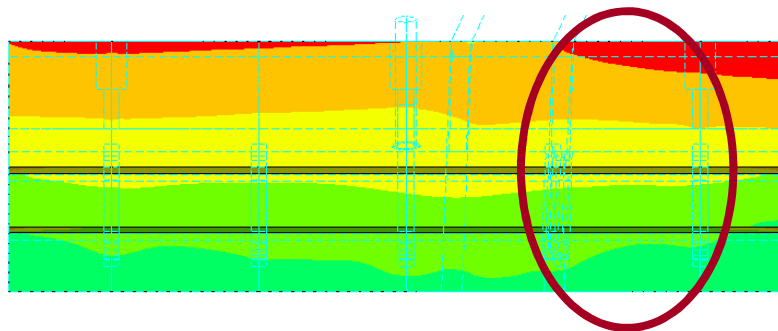


$$k_{red} = 203.69, \lambda_{red} = 35.194$$

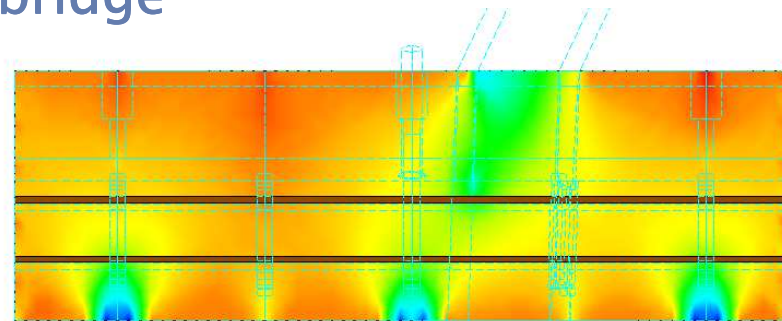
A) TYPICAL  
PLATES  
energy method



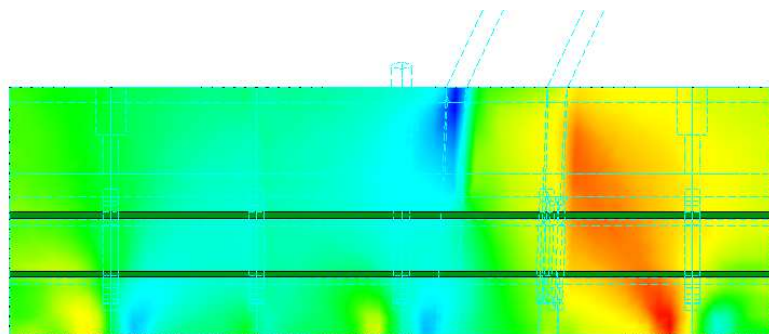
## Actual calculations on the Danube bridge



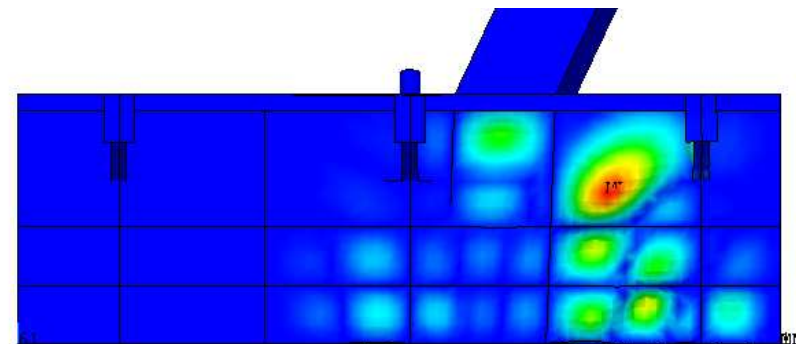
axial stresses [MPa]



vertical stresses [MPa]



shear stresses [MPa]



$$\alpha_{cr} = 8.332$$

buckling shape

(3) Irregular configuration and stress field – ??? no rule given

- assume plate-type behaviour → generalized
- plate slenderness ( $\lambda_0$ ) and reduction factor ( $\phi_b$ )

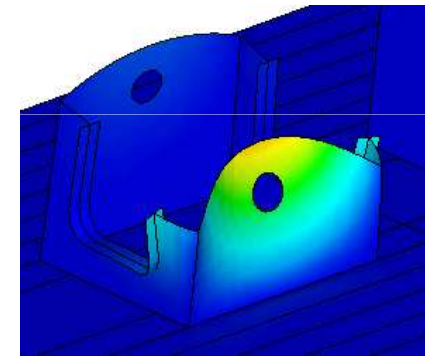
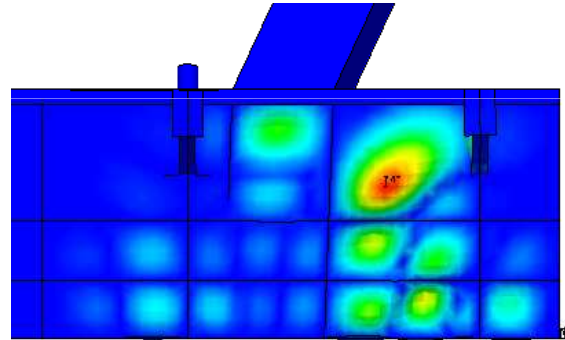
$$\sigma_{red,cr} = \alpha_{cr} \sigma_{red,max}$$

$\alpha_{cr}$ : critical load factor

$$\lambda_0 = \sqrt{\frac{\pi^2 E}{\sigma_{red,cr}}}$$

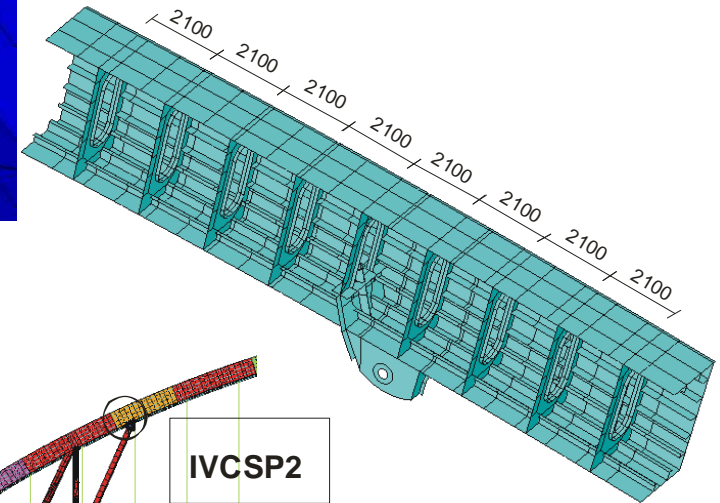
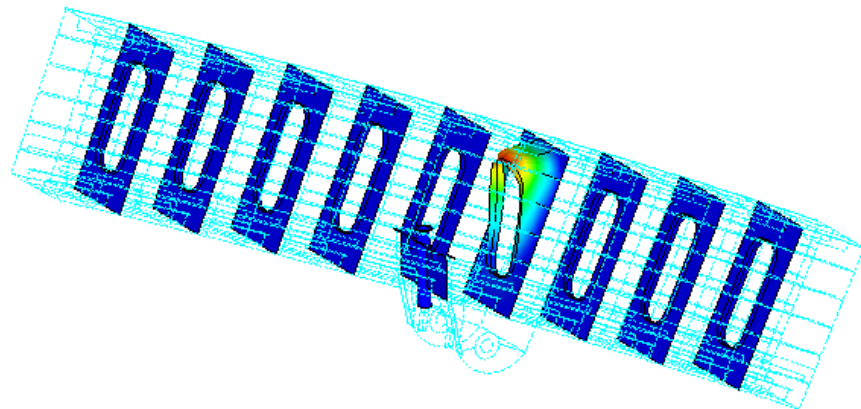
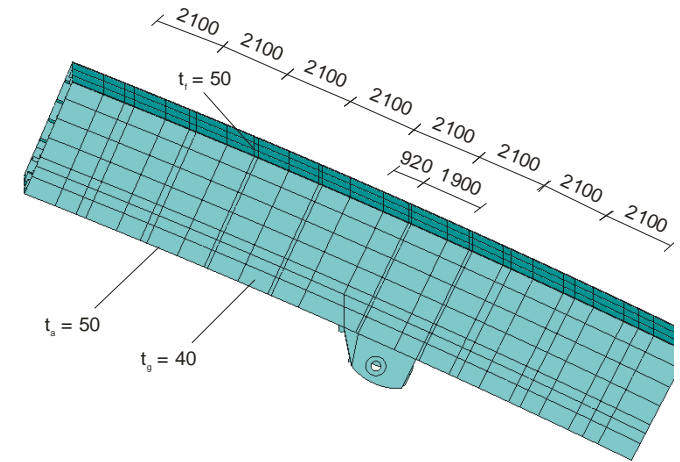
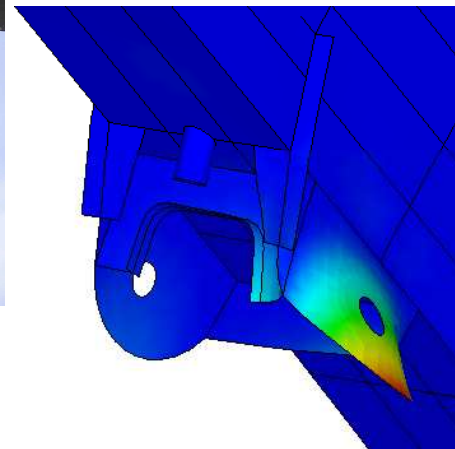


$\phi_b$  reduction factor for plates



• check:  $\sigma_{red} = \sqrt{\sigma^2 + 3\tau^2} \leq \phi_b \cdot \sigma_e$

## Actual calculations on the Danube bridge







## Design methods – Eurocode 3 Part 1–5

- (1) basic procedure for stiffened plates in complex stress fields (no use of numerical models)
- (2) partial use of FEM: plate slenderness from bifurcation analysis
- (3) reduced stress method
- (4) finite element analysis based design (full numerical simulation)



## (1) Basic procedure (no use of numerical models)

- consideration of both plate-type and column-like buckling

a) plate-type:

$$\lambda_p = \sqrt{\frac{\beta_{A,c} f_y}{\sigma_{cr,p}}}$$

 $\rho$  $\sigma_{cr,p}$  crit. stress for overall buckling  
e.g. from orthotropic plate theory

b) column-like:

$$\lambda_c = \sqrt{\frac{\beta_{A,c} f_y}{\sigma_{cr,c}}}$$

 $\chi_c$ 

$$\sigma_{cr,c} = \frac{\pi^2 EI_{sl,1}}{A_{sl,1} a^2}$$

c) interpolation:  $\rho_c = (\rho - \chi_c)\xi(2 - \xi) + \chi_c$   $\xi = \frac{\sigma_{cr,p}}{\sigma_{cr,c}} - 1$  ( $0 \leq \xi \leq 1$ )

- cross-section resistance – with effective area

$$N_{c,Rd} = \frac{A_{c,eff} f_y}{\gamma_{M0}}$$

• check:  $N_{Ed} \leq N_{c,Rd}$

(1) Basic procedure (no use of numerical models)

- consideration of both plate-type and column-like buckling

a) plate-type:

$$\lambda_p = \sqrt{\frac{\beta_{A,c} f_y}{\sigma_{cr,p}}}$$

↓  
 $\rho$

$\sigma_{cr,p}$  crit. stre  
e.g. from

b) column-like:

- proposal for modification (Maquoi, Skaloud):

$$\rho_c = \chi_c \text{ but not smaller than } \rho_{p,\infty}$$

$$\frac{2EI_{sl,1}}{\lambda_{sl,1}^2 a^2}$$

c) interpolation:

$$\rho_c = (\rho - \chi_c)\xi(2 - \xi) + \chi_c$$

$$\xi = \frac{\sigma_{cr,p}}{\sigma_{cr,c}} - 1 \quad (0 \leq \xi \leq 1)$$

- cross-section resistance - with effective area

$$N_{c,Rd} = \frac{A_{c,eff} f_y}{\gamma_{M0}}$$

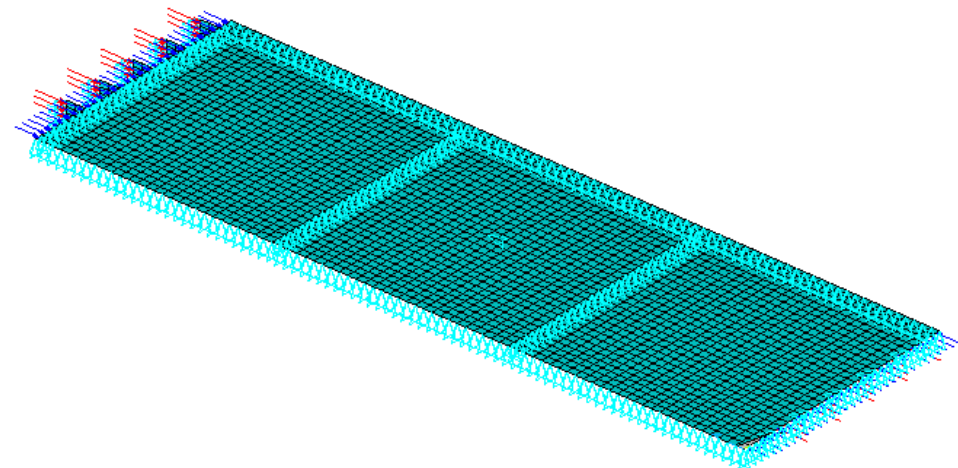
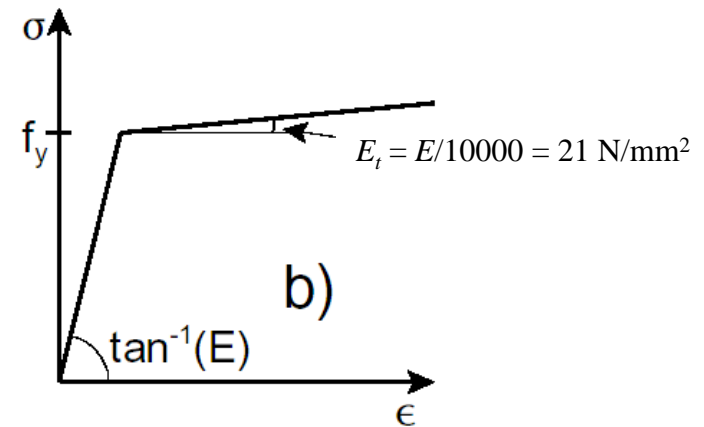
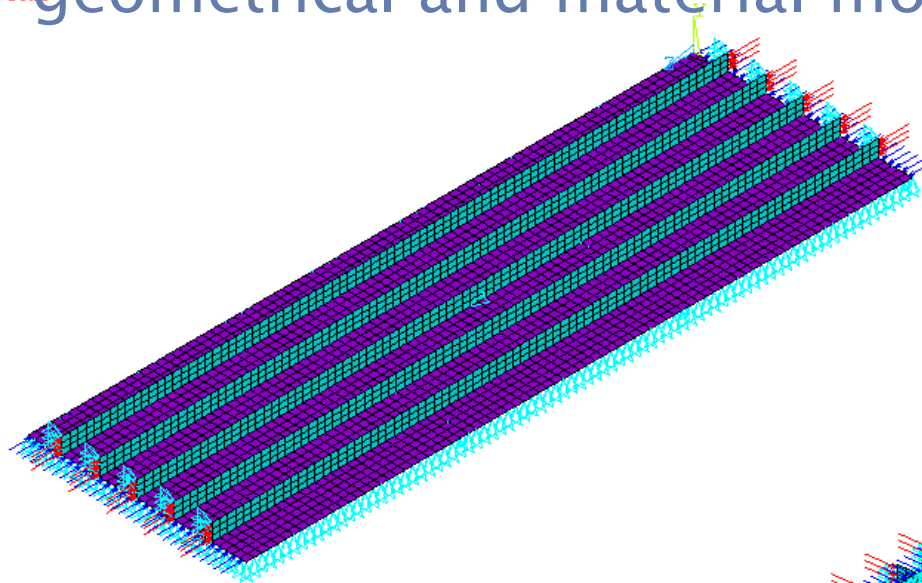
- check:  $N_{Ed} \leq N_{c,Rd}$

#### (4) Finite element analysis based design

- geometrical and material non-linearity
- equivalent geometric imperfections
- non-linear simulation



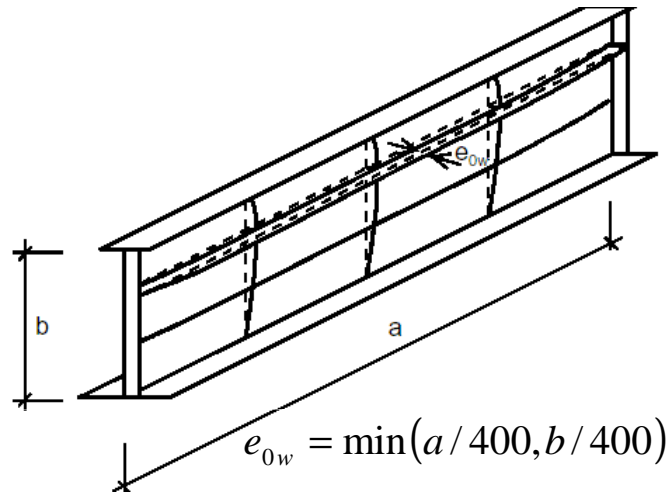
• geometrical and material model



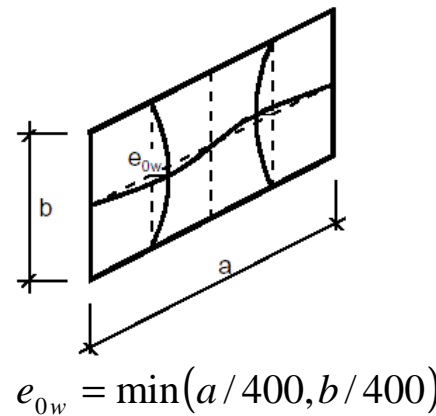
Case Nr.	$t_p$ [mm]	$b$ [m]	$a$ [m]	stiffener
1	40	2	4.56	2 x 280-22
2	30	3.8	4.56	5 x 280-22
3	50	2	2.125	2 x T270-150-22
4	20	3.8	3.9	5 x 280-22
5	16	3.8	3.86	5 x 280-22
6	20	2	3.86	2 x 280-22

$t_p$  – plate thickness;  $b$  – plate width;  $a$  – plate length between transverse stiffeners or diaphragms

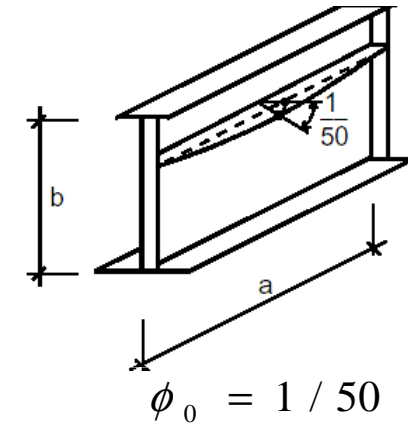
• equivalent geometric imperfections



a) global imperfection of stiffener



b) imperfection of subpanel



c) local imperfection of stiffener

~ alternatively, relevant buckling shapes, i.e.

- a) overall buckling,
- b) local buckling of subpanels,
- c) torsion mode of the stiffener



- equivalent geometric imperfections

combination of the imperfections:

leading (100%) + others (70%)



**PROBLEM** when using  
buckling shapes as imperfections:

overall/local plate buckling usually  
accompanied by the torsion of stiffener

the requirements for the imperfection  
amplitudes are difficult to satisfy

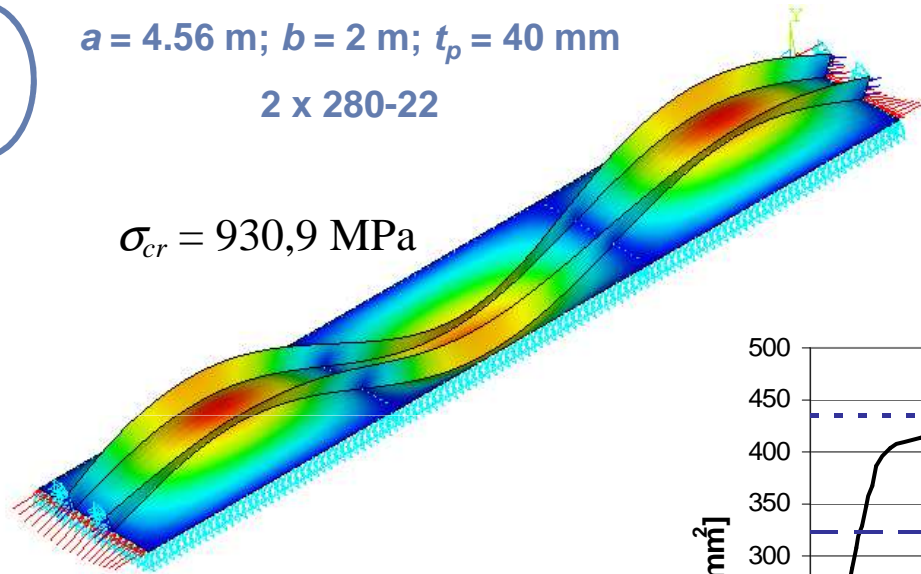
## Actual calculations on the Danube bridge

1

$a = 4.56 \text{ m}; b = 2 \text{ m}; t_p = 40 \text{ mm}$

2 x 280-22

$\sigma_{cr} = 930,9 \text{ MPa}$

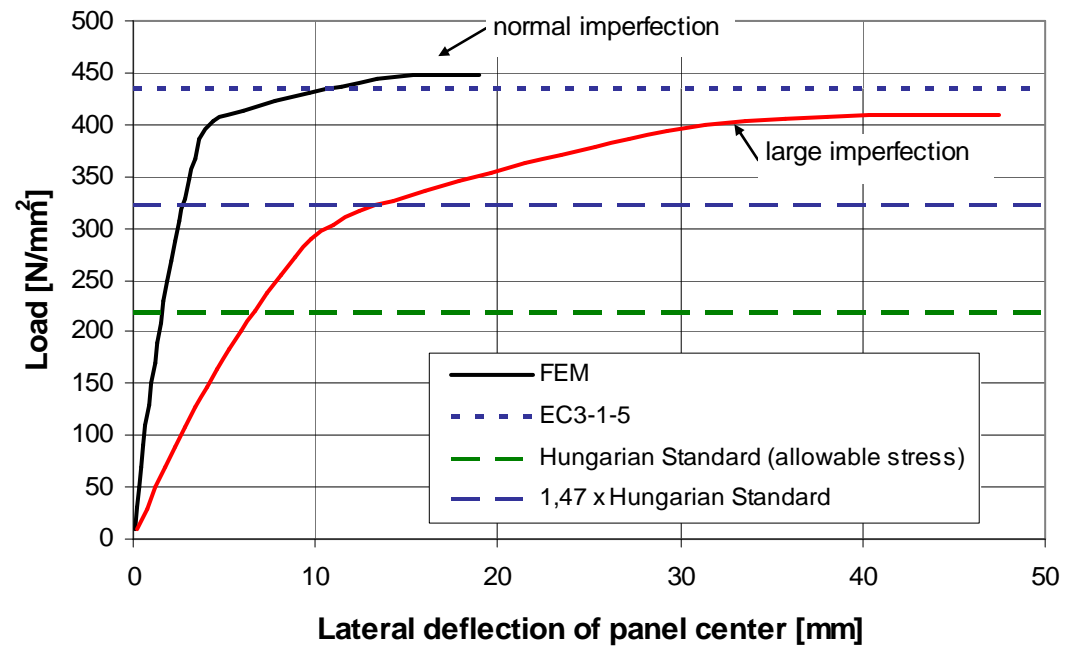


$$e_{0w,1} = \min(a/400, b/400) = 2000/400 = 5 \text{ mm}$$

$$e_{0x,2} = h_b \phi_0 = 280 \cdot \frac{1}{50} = 5,6 \text{ mm}$$

$$\alpha_{0,1} = \frac{5,00}{2,588 \cdot 10^{-3}} = 1932$$

$$\alpha_{0,2} = \frac{5,6}{0,661 \cdot 10^{-3}} = 8472$$

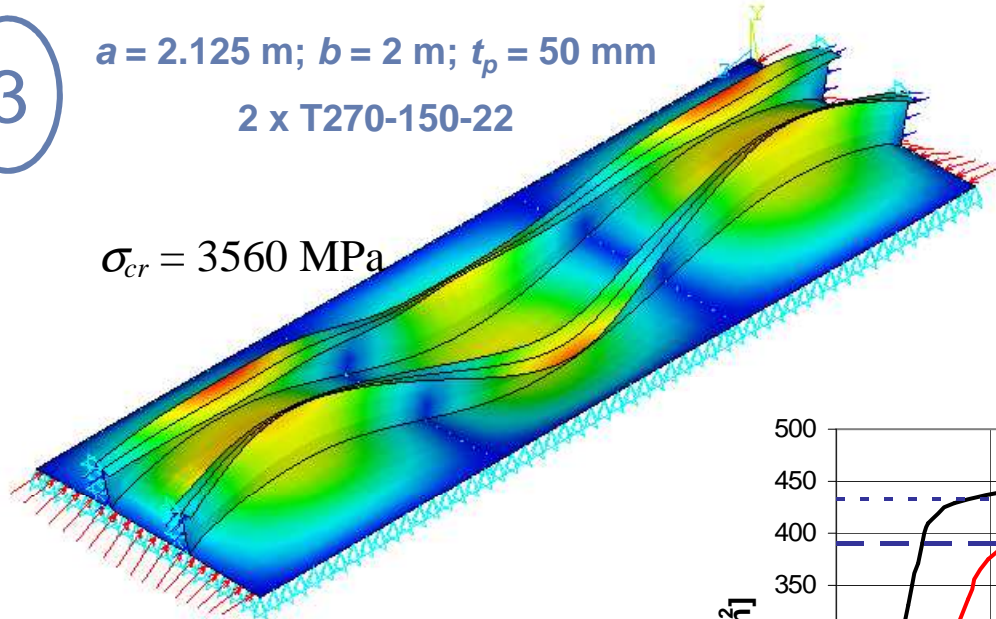




3

$a = 2.125 \text{ m}; b = 2 \text{ m}; t_p = 50 \text{ mm}$   
 2 x T270-150-22

$\sigma_{cr} = 3560 \text{ MPa}$

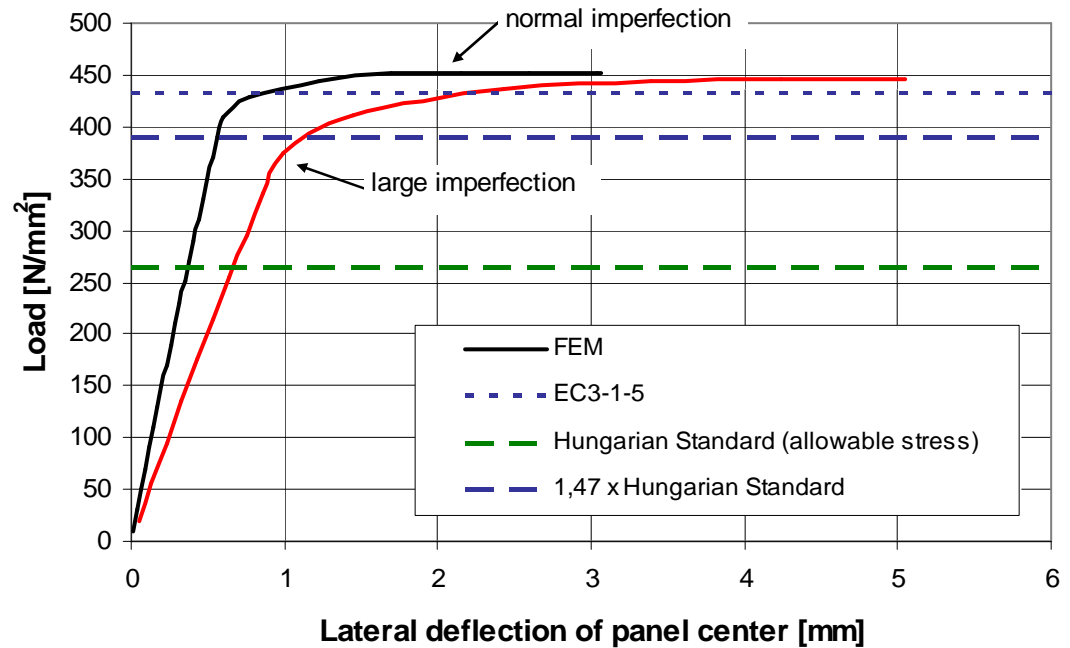


$$e_{0w,1} = \min(a/400, b/400) = 2000/400 = 5 \text{ mm}$$

$$e_{0x,2} = h_b \phi_0 = 270 \cdot \frac{1}{50} = 5,4 \text{ mm}$$

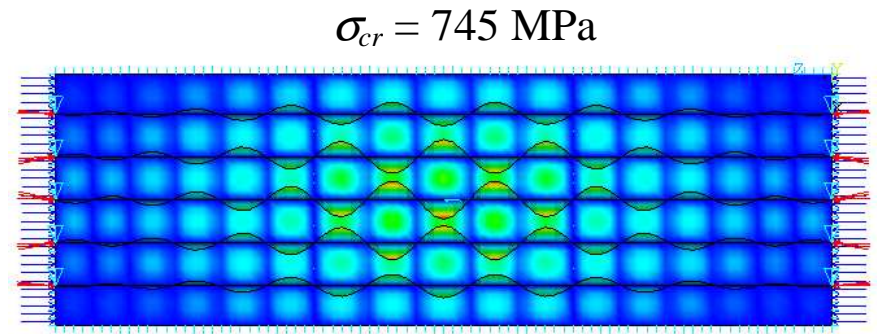
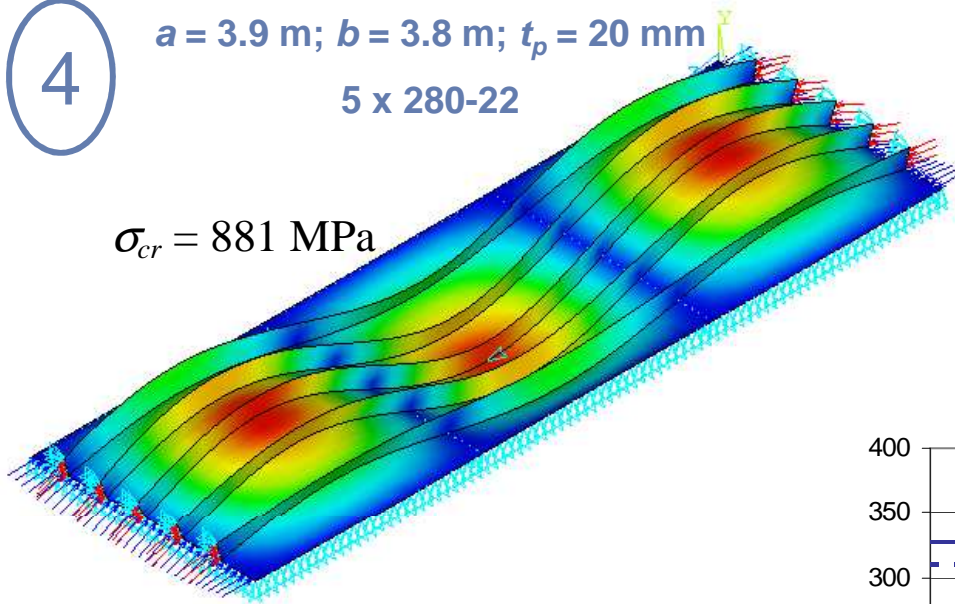
$$\alpha_{0,1} = \frac{5,00}{0,756 \cdot 10^{-3}} = 6614$$

$$\alpha_{0,2} = \frac{5,4}{0,445 \cdot 10^{-3}} = 12135$$



4

$a = 3.9 \text{ m}; b = 3.8 \text{ m}; t_p = 20 \text{ mm}$   
 5 x 280-22



$$e_{0w,1} = \min(a/400, b/400) = 3800/400 = 9,5 \text{ mm}$$

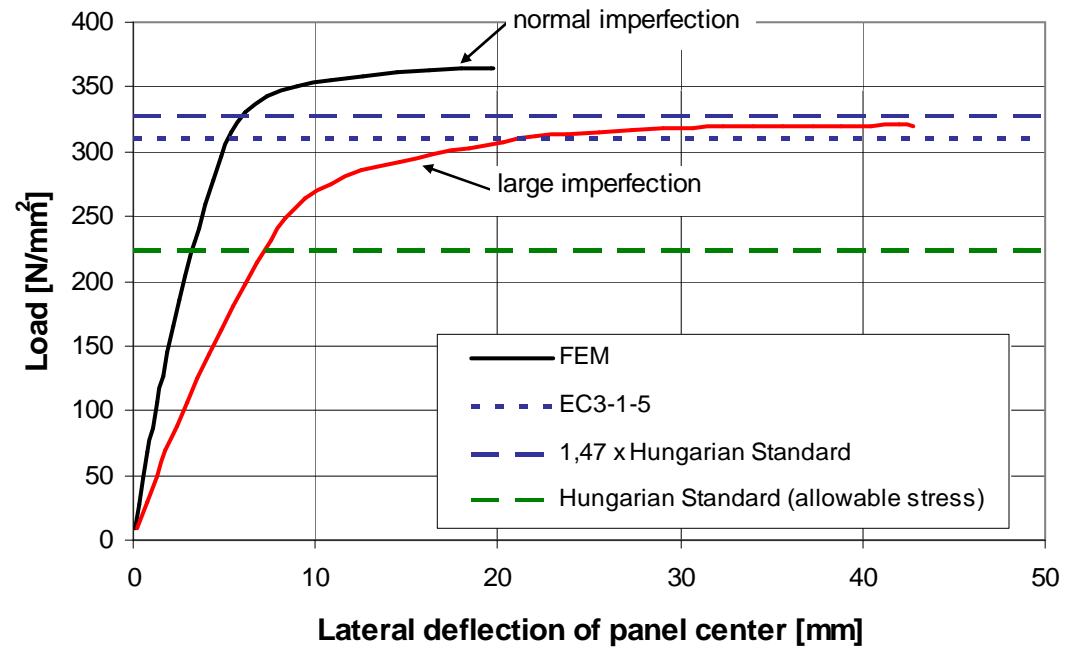
$$e_{0x,2} = h_b \phi_0 = 280 \cdot \frac{1}{50} = 5,6 \text{ mm}$$

$$e_{0w,3} = 650/400 = 1,625 \text{ mm}$$

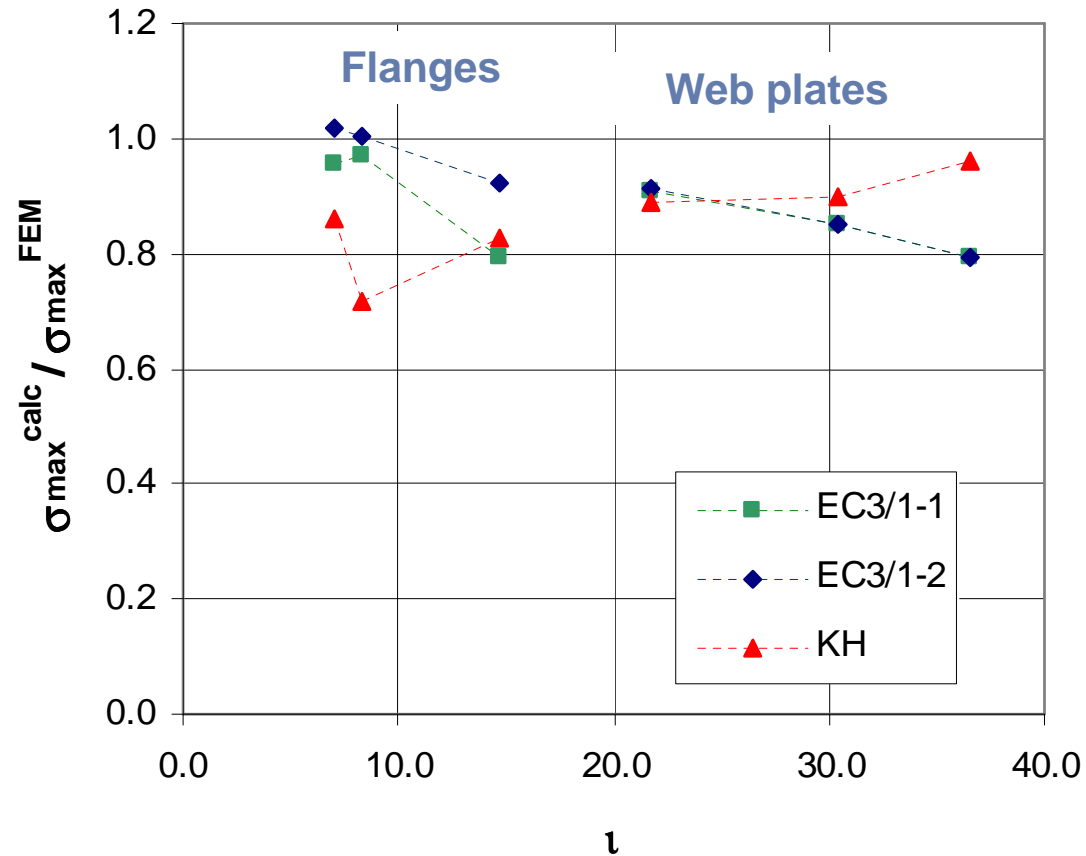
$$\alpha_{0,1} = \frac{9,5}{2,278 \cdot 10^{-3}} = 4170$$

$$\alpha_{0,2} = \frac{5,6}{0,577 \cdot 10^{-3}} = 9705$$

$$\alpha_{0,3} = \frac{1,625}{0,813 \cdot 10^{-3}} = 1999$$



# Comparison



$$\lambda = \frac{\sqrt{\delta} b_f}{\sqrt[4]{\gamma} t_f}$$



# Concluding remarks

## Tied arch bridge project

### Studies on global stability of tied arch

- Model test
- Evaluation of classical and advanced design methods in comparison to the test ultimate loads.

### Studies on the buckling of orthotropic plates

- Design methods – classical and advanced
- Comparison of different design methods to FE simulation based results.

## Application



Budapest University of Technology and Economics



# DEPARTMENT OF STRUCTURAL ENGINEERING



# Thank you for your attention!